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DEVELOPMENT OF A TWO-LAYER FLUID FLOW NEAR A ROTATING DISK UPON ACCELERATION

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We consider the problem of the development of boundary-layer flow on a suddenly accelerated disk in a two-layer fluid when at the initial instant of time the interface between the fluids is perpendicular to the plane of the disk.

The efficiency of certain processes of chemical technology, such as centrifugal extraction, depends on the boundary-layer flow of fluids over a disk rotating about its axis during a change in its angular velocity [1]. It is assumed that the two phases of an incompressible fluid are not mixed and differ in density (ρ_1, ρ_2) and kinematic viscosity (ν_1, ν_2). In the case of prolonged rotation of a container with a constant angular velocity Ω_0, the phase interface is vertical, $f(z) \equiv r^* \equiv$ const, and all of the velocity vector components in the coordinate system r, z, θ fixed on the disk are equal to zero: $V_r = V_z = V_\theta = 0$. When at the time moment $\tau = 0$ the speed of rotation increases, $\Omega = \Omega_0 + \omega$, near the surface $z = 0$, a radial flow (the Ekman layer) develops, which deforms the phase interface: $r = f(z, \tau)$. The latter leads to the appearance of buoyancy forces, which stabilize the position of the interface. For a homogeneous fluid ($\rho_1 = \rho_2$, $\nu_1 = \nu_2$) this class of flows has been investigated rather extensively [2-4]. However, until recently not enough attention has been devoted to theoretical investigations of the dynamics of a two-layer fluid. In this article we present the results of a mathematical and an empirical modeling of the acceleration process.

A preliminary analysis of the initial Navier–Stokes equations from the standpoint of similarity theory [5] showed that at small values of the Rossby, $\varepsilon = \omega/\Omega_0$, and Ekman, $\delta = \sqrt{\nu_1 \Omega_0}/(\omega r^*)$, numbers the flow in the Ekman layer can be described within the scope of a simplified mathematical model:

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial y} = 2V + \frac{\partial}{\partial y}\left(K_\nu \frac{\partial U}{\partial y}\right) + BK_\rho H, \tag{1}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + W\frac{\partial V}{\partial y} = -2U + \frac{\partial}{\partial y}\left(K_\nu \frac{\partial V}{\partial y}\right), \tag{2}$$

$$\frac{\partial U}{\partial x} + \frac{\partial W}{\partial y} = 0, \quad -\infty < x < +\infty, \quad y > 0, \tag{3}$$

$$x = F(y, t): \frac{\partial F}{\partial t} = U - W\frac{\partial F}{\partial y}, \quad F(0, t) = F(y, 0) = 0, \tag{4}$$

$$|x| \to \infty: W = 0 \quad \frac{\partial U}{\partial x} = \frac{\partial V}{\partial x} = 0, \tag{5}$$

$$y = 0: U = W = 0, \quad V = 1; \quad y \to \infty: U = W = 0, \tag{6}$$

$$t = 0: U = V = W = 0, \tag{7}$$

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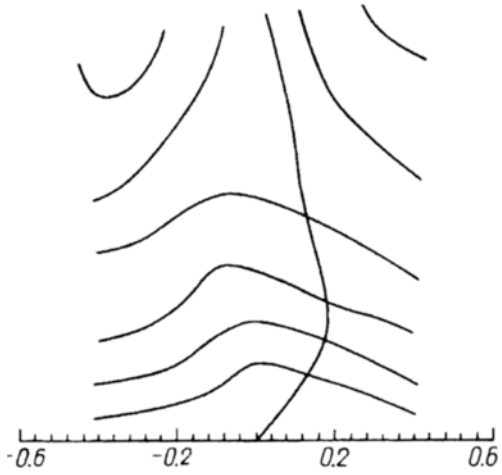

Fig. 1. Streamlines and phase interface at $A = 1$ and $t = 2$.

where

$$H = \begin{cases} 1, & F < x < 0, \\ -1, & 0 < x < F, \\ 0, & \text{otherwise} \end{cases} \quad K_\nu = \begin{cases} 1, & x < F, \\ \dfrac{\nu_2}{\nu_1}, & x > F, \end{cases} \quad K_\rho = \begin{cases} 1, & x < F, \\ \dfrac{\rho_1}{\rho_2}, & x > F. \end{cases} \tag{8}$$

Problem (1)-(8) is written in dimensionless variables

$$t = \tau\Omega_0, \quad x = \frac{r - r^*}{\varepsilon r^*}, \quad y = \frac{z\delta^2}{r^*}, \quad F = \frac{f - r^*}{\varepsilon r^*},$$

$$U = \frac{V_r}{\omega r^*}, \quad V = \frac{V_\theta}{\omega r^*}, \quad W = \frac{V_z}{\delta\omega r^*}, \quad B = \frac{\rho_2 - \rho_1}{\varepsilon\rho_1}, \tag{9}$$

with terms of the order of $0(\varepsilon + \delta^2)$ being discarded. In deriving equations (1)-(2) it was assumed that viscosities ν_1 and ν_2 were of the same order of magnitude. The mathematical model obtained differs from the well-known ones [2, 3] by the term $BK_\rho H$ in Eq. (1) and is reduced to them at $B = 0$.

For a numerical realization of model (1)-(9) a finite-difference method is suggested, which is described in detail in [5]. When passing to a new time layer, the functions U, V, W, and F are found with the aid of an iterative process; the criterion cessation of iterations is stabilization of the position of the boundary F. In each iteration the difference analogs of Eqs. (1)-(3) are solved in succession to determine velocities V, U, and W, as is kinematic Eq. (4), which makes it possible to determine the position of the boundary F and to determine more precisely the coefficients H, K_ρ, and K_ν by formulas (8). To ensure stability of the calculation algorithm, the convective terms in Eqs. (1) and (2) were approximated with the aid of "counterflow differences," while an artificial viscosity in the x direction was introduced into Eqs. (1) and (3). The coefficients of pseudoviscosity were selected in accordance with the steps of a rectangular grid. This provided the first order of the difference scheme approximation [6]. The boundaries of the grid region that simulate infinity in boundary conditions (5) and (6) were determined experimentally from the requirement that their further removal from the coordinate origin does not influence the solution of the difference scheme.

The calculation algorithm was tested by the method of enhanced grid resolution. We determined the parameter values that provided acceptable calculation accuracy. The mean calculation time within the interval $0 < t < 3$ with a time step of $5 \cdot 10^{-3}$ on a grid containing 50×40 nodes along the x and y coordinates is 60 min on an ES-1045 computer, including the calculation of streamlines and the derivation of graphical information. Comparison of the calculated results for the dynamics of a homogeneous fluid ($K_\rho = 1$, $K_\nu = 1$, $B = 0$) with a

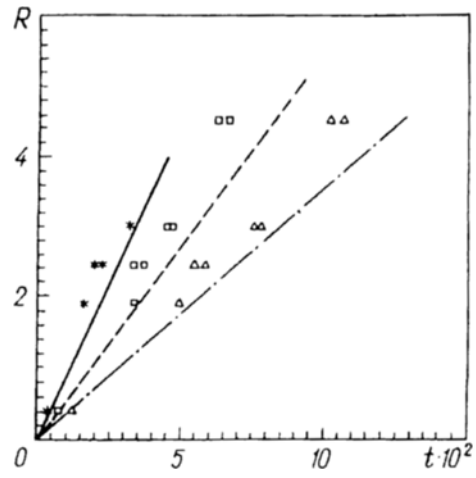

Fig. 2. Comparison of experimental and predicted results on the mutual penetration of phases. R, mm.

well-known precise solution [3] showed that the maximum deviation did not exceed 5%, with the numerical solution reflecting all the qualitative aspects of the analytical one, including the presence of weak reverse flows.

As seen from the statement of problem (1)-(8), the process of acceleration is determined by two dimensionless groups: $A = B\rho_1/\rho_2$ and ν_2/ν_1. Numerical experiments showed that a change in the ratio ν_2/ν_1 within the range of 0.5–2 exerts an insignificant effect on the pattern of fluid flow and the depth of the mutual penetration of phases, whereas the effect of the Archimedian force A is predominant. A typical pattern of streamlines in the xOy plane and the characteristic shape of the phase interface are shown in Fig. 1. An important flow feature when $0 < A < 2$ is the fact that appreciabele penetration of the light phase into the heavy phase proceeds up to the moment $t \approx 2$ and then stops. In this case an ascending flow along the line $x = 0$ is formed that does not enter the zone of phase substitution. It is found that when $A \geq 2$ the penetration of one fluid into the other becomes insignificant (less than $0.03\varepsilon r*$), which is important for problems of chemical technology, for example, mass transfer.

Along with mathematical simulation of the acceleration process, we carried out experimental investigations. Since it is virtually impossible to measure the local velocity fields in a boundary layer, because of its small dimensions (the Ekman layer $\approx$ 0.3 mm), and also in view of the fact that the magnitude of the interphase surface area is of independent interest for chemical technology, emphasis was placed on determination of the dynamics of the boundary F. Investigations were carried out in an experimental cylindrical container with an annular channel with a cross-section of 40 × 40 mm. The channel was filled with two fluids: a lighter electrically conducting one (water + NaCl; $\rho_1 = 1001$ kg/m^3, $\nu_1 = 0.94 \cdot 10^{-6}$ m^2/sec) and a heavier one (CCl_4 + kerosene; $\rho_2 = 1120$ kg/m^3; $\nu_2 = 1.6 \cdot 10^{-6}$ m^2/sec). To measure the depth of penetration of the light phase in the radial section of the channel we installed seven electrodes at distances 95, 100, 101.9, 102.4, 103, 104.1, and 104.8 mm from the axis of rotation. The first of these (anode) was immersed in the electrically conducting fluid. The second one (cathode) was used to control the position of the phase interface in steady rotation.

During acceleration, the cathodes were alternately closed by a layer of the penetrating electrically conducting fluid, and the corresponding signals were recorded by an NO71.2 light-beam oscillograph. The container was rotated by two motors, the switching of which made it possible to increase the rotation speed from $\Omega_0 = 107$ sec^{-1} to 118, 125, and 137 sec^{-1} for a time interval not exceeding 0.25 sec. The rotation frequency was controlled by a TE-204 tachometer whose readings were recorded by the same light-beam oscillograph.

We carried out three experiments that corresponded to different values of the criterion A. The results of measurements of the maximum advance of the phase interface are shown in Fig. 2. The lines designate the results of numerical calculation (the solid line corresponds to $A = 1.04$, the dashed line to $A = 0.64$, and the dashed-dotted line to $A = 0.35$). The points represent experimental data. The discrepancy of the results does not exceed 15%. This is compatible with the uncertainty of both the experiment itself and the mathematical model adopted.

In conclusion we note that the model (1)-(8) does not take into account the effect of the forces of intermolecular tension. The satisfactory agreement between the results of simulation and experiment allows it to be assumed that the effect of the indicated forces is localized in the neighborhood of the end of the penetrated phase, where the curvature radius is fairly small.

NOTATION

ρ_1, ρ_2, densities of fluids (kg/m); ν_1, ν_2, kinematic viscosities of fluids (kg/m); Ω_0, initial angular velocity (1/sec); (r, z, θ), cylindrical coordinate system; V_r, V_z, V_θ, fluid velocity components relative the rotor (m/sec); τ, time (sec); ω, value of change in the angular velocity (1/sec); t, dimensionless time; U, V, W, dimensionless velocity components; $\varepsilon = \omega/\Omega_0$, Rossby number; $\delta = \sqrt{\nu_1\Omega_0}/(\omega r*)$, Ekman number; $B = (\rho_2 - \rho_1)/(\varepsilon\rho_1)$; $A = B\rho_1/\rho_2$.

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